

Equation Equivalence

Context

Two of my contributions ([1], [2]) use two different equation forms for the Engset Loss Model blocking probability. Here, we show that they are the same.

Equation Forms

The correct version of the blocking probability P in Equation 8 [1] is given by our Errata [3]:

$$P = \frac{\binom{S-1}{m} \rho^m}{\sum_{i=0}^m \binom{S-1}{i} \rho^i}, \quad \rho = \frac{1/\mu}{1/\lambda_B} = \frac{\lambda_B}{\mu} \quad (1)$$

The version used in [2] is:

$$P = \frac{\binom{S-1}{m} A^m}{\sum_{i=0}^m \binom{S-1}{i} A^i}, \quad A = \frac{\alpha}{1 - \alpha(1 - P)} \quad (2)$$

where A is the busy period arrival rate and α is the *offered traffic per source*.

Equivalence

Here, we show that Eq(1) and Eq(2) are equivalent by showing that $\rho = A$, that is:

$$\frac{\lambda_B}{\mu} = \frac{\alpha}{1 - \alpha(1 - P)} \quad (3)$$

In [1], the busy period arrival rate was given by Equation 6 and is reproduced here:

$$A = S \left(\frac{1}{\frac{1}{\lambda_B} + (1 - P)\frac{1}{\mu}} \right)$$

By Little's Law, the *total offered traffic* is the arrival rate multiplied by the mean duration of each request. The offered traffic per source is the total offered traffic divided by the number of sources S :

$$\text{total offered traffic} = S \left(\frac{\frac{1}{\mu}}{\frac{1}{\lambda_B} + (1 - P)\frac{1}{\mu}} \right) \quad (4)$$

$$\text{offered traffic per source } \alpha = \frac{1}{\frac{\mu}{\lambda_B} + 1 - P} \quad (5)$$

Finally, we have

$$\left(\frac{\mu}{\lambda_B} + (1 - P)\frac{1}{\mu} \right) \alpha = 1 \quad (6)$$

$$\frac{\mu}{\lambda_B} = \frac{1 - \alpha(1 - P)}{\alpha} \quad (7)$$

$$\frac{\lambda_B}{\mu} = \frac{\alpha}{1 - \alpha(1 - P)} \quad (8)$$

And the desired result of Eq(3) is achieved.

References

- [1] T. Carpenter, S. Keshav, and J. Wong, "Sizing finite-population vehicle pools," *IEEE Transactions on Intelligent Transportation Systems*, vol. PP, no. 99, pp. 1–11, 2014.
- [2] P. Azimzadeh and T. Carpenter, "Fast engset computation," *Operations Research Letters*, vol. 44, no. 3, pp. 313 – 318, 2016.
- [3] http://tommyjcarpenter.com/papers/2014/car_pools_errata.pdf.