The Impact of Electricity Pricing Schemes on Storage Adoption In Ontario

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ABSTRACT
The Ontario electrical grid is sized to meet peak electricity load. If this worst-case load were reduced, the government and Ontario taxpayers could defer large infrastructural costs, reducing the cost of generation and electricity prices. Storage, batteries that can store energy during times of low load and be discharged during times of peak load, is one proposed solution to reducing peak load. We evaluate the effect of storage on the electrical grid under different customer electricity pricing schemes. We find that for existing pricing schemes, adopting storage is not profitable. Furthermore, as the level of storage adoption in the population increases, pricing schemes that incentivize charging at times known to all homeowners will eventually increase the peak load rather than decrease it. However, in some circumstances particular levels of homeowner storage adoption helps the grid reduce peak load, and thus the grid may choose to subsidize the cost of storage. We discuss hypothetical pricing schemes under which storage adoption is profitable for homeowners.

1. INTRODUCTION
Annual peak load is the highest aggregate demand for electricity throughout the year. Daily peaks are defined similarly. Ontario electricity market, from generators to customers, stand to save money from the reduction of peak load. Therefore there are large initiatives to reduce peak load [7,16,19,22,24]—the government is willing to spend $12 billion over the next 30 years in doing so [25].

There are two main ways people can reduce their peak load, both of which are envisioned in the coming “smart grid” [3,5,17,18]. People can store energy during off peak periods (in addition to fulfilling their normal loads), and use it to fulfill some of their load during peak periods. People can also change the time of their loads (for example, by doing laundry in the late evening), known as demand response. We focus exclusively on storage. It is less intrusive, as consumers are not required to modify their routines. However, the adoption of storage comes at a monetary cost to the consumer.

Storage can be profitable: if the price of electricity during peak periods is much higher than off peak periods, homeowners can recuperate the cost of storage by saving on their electricity bills. However, if people cannot recuperate this cost due to a poorly chosen electricity pricing scheme, people will not adopt storage. Moreover, with the wrong pricing scheme, storage owners can actually increase peak load rather than decrease it. Our goal is to compare the profitability of storage under different pricing schemes and to study its effects in reducing the peak load.

Specifically, our main contributions are:

1. We use measured home load data to compare the effects of different pricing schemes on the Ontario grid given various homeowner storage penetration rates—the proportion of homeowners with storage. We show that under high storage penetration, pricing schemes that incentivize charging at times known to all homeowners can increase peak load rather than decrease it and explain this counterintuitive result.
2. We find that storage is not profitable under Ontario’s current pricing schemes, but present hypothetical pricing schemes under which storage is profitable.
3. We provide a simulation methodology and an optimization model for homeowners to optimize their storage profiles.

2. RELATED WORK
Storage has been traditionally too expensive for homeowners to adopt, so there has been limited work in this area. The closest related work to ours is that of Vytilingum et al. [31]. They provide an optimization model for homeowners with storage, to compute their best response storage profiles to a hypothetical pricing scheme. We modify their optimization model as follows. First, we remove several simplifying assumptions. Second, we use measured data from real homes rather than simulating test data. Finally, the profitability analysis used by Vytilingum et al. does not consider the initial cost for homeowners to purchase storage. We give prices for storage and consider the cost of battery degradation and battery replacement costs in our profitability analysis.

There have been three other research efforts in this area. Daryan-nian et al. [6] provide an optimization algorithm for homeowners’ best storage responses to electricity spot prices. Vytilingum et al. expand upon this. Houwing et al. [11] give algorithms for homeowners to optimize their control of combined heat and power (CHP) systems, but they consider algorithms for each individual home and do not consider the effect of many households optimizing their CHP on the grid. Exarchakos at al. [8] consider the profitability of storage for homeowners when their storage is externally controlled (by a demand response program) to create savings by storing energy when it is inexpensive and discharging when it is expensive. Our conclusions match theirs; storage is not profitable
for homeowners if the difference between peak and off peak prices is not significant. However, they only consider each home individually maximizing their profit and do not consider the effect of many homes using storage on the grid. As mentioned, the Ontario government is willing to spend billions of dollars on reducing peak load and may opt to subsidize storage if it helps accomplish this goal. Alternatively, they may have incentives to change their electricity pricing scheme to make storage profitable for consumers and to encourage charging during times that benefit the grid. Thus there is still a need to study the effect on the grid of many homeowners adopting storage.

3. MOTIVATION

In this section, we describe why reducing peak load benefits everyone in Ontario’s electricity market. We begin with a brief overview of Ontario’s electricity market in Section 3.1, and explain why higher peak load induces a higher price on everyone in the system in Section 3.2. In Section 3.3, we give a brief summary of storage pricing. Given this background, in Section 3.4 we formally define selecting a pricing scheme that most incentivizes the use of residential storage, both to save adopters money and to reduce peak load, as a mechanism design problem.

3.1 Ontario Energy Market Overview

The Ontario energy market is divided into two simultaneously operating submarkets, which we refer to as the front-end and back-end markets. We briefly describe each.

The Independent Electricity System Operator (IESO) coordinates the back-end market by matching supply and demand. Every day, the IESO predicts the aggregate demand for each hour of the following day [14]. Generators then bid to generate portions of this demand, after which the IESO selects a set of bids that minimizes the cost of fulfilling the demand [33]. As actual demand differs from day-ahead predictions, each prediction for hour h is refined twice before bids are finalized—an hour before h and then again 5 minutes before h. Collectively, these three auctions are known as the “day ahead”, “hour ahead”, and “5-minute ahead” auctions.

Participants in the back-end market are grouped into two categories: dispatchable and nondispatchable. Dispatchable participants are generators that bid in the auctions discussed above. Nondispatchable participants are further divided into two categories. The first category, composed of large industrial loads and distributors, buy wholesale electricity for the Hourly Ontario Energy Price (HOEP) [33], determined by the IESO. The second category consists of renewable generating facilities that bid to generate at the HOEP. These facilities are not allowed to participate in the auctions because they are not reliable (e.g. wind power), and the IESO must ensure that demand is satisfied.

Consumers that do not buy enough energy to buy wholesale, such as homeowners and small businesses, make up the front-end market. Currently in Ontario, these consumers are shielded from the volatile pricing that takes place in the back-end market by distribution companies, which provide a bridge between the two submarkets. Distribution companies sell energy to consumers using pricing schemes determined by the Ontario Energy Board (OEB) under the Regulated Pricing Plan (RPP) [21]. Depending on the eligibility of the end-consumer, these prices can be determined under the Tiered Usage-Based Pricing or Time of Day Pricing schemes, described in Sections 6.1 and 6.2 respectively.

Every six months, the RPP is reevaluated and changed according to the difference in the amount paid by the consumer and the amount paid to generators [27]. If generation costs change significantly in either direction, the RPP will change accordingly.

3.2 Load Factor

The primary measure of “peakedness” of load is load factor [32]. Load factor is defined as peak load divided by average load. There are two main reasons why reducing the load factor benefits both the grid and residents. First, the grid is sized to support worst case load. Figure 1 shows the Ontario load curve, a sorted curve of hourly aggregate load plotted against the frequency of that load, for each year between 2003 - 2009 [12]. We can see the worst case loads occur for less than 1% of each year. If these peaks were reduced, the grid could eliminate large impending infrastructure costs, which saves both the government and taxpayers money. As an estimate, the Ontario government states 70% of generation infrastructure needs to be replaced in the next 20 years, totaling $87 billion required from public and private investment [22]; since the grid is sized for worst case peak demand, reducing the peak by 1% saves roughly $870 million in costs!

The other incentive to reduce the load factor is that the cost to generate load is not linear as load increases. Figure 2 shows the relationship between the HOEP and aggregate load on a representative day [12]. We can see as aggregate load increases, the cost to generate per-unit ($/Mw) increases. The reason is base load power plants, which in Ontario are largely composed of nuclear and hy-
dro, have low fuel costs but cannot be turned on and off rapidly and thus cannot fulfill fluctuating peak demands [28]. Peaking power plants, usually natural gas and oil fired plants, have a higher cost of generation but can be turned on and off quickly to fulfill peak demand [29]. Reducing peak load reduces the amount of power generated from these more expensive plants, which decreases the generators’ bids to produce electricity. If these bids drop significantly, then the RPP will also be eventually reduced, thus benefiting customers and the grid.

### 3.3 Storage

Storage is used in a variety of applications, from portable electronics to batteries powering electric vehicles (EVs). Batteries used in different settings have different costs and lifetimes, and there is a tradeoff between the two. For example, modern day EV batteries can go through thousands of cycles and still maintain 95% of original charging capacity [30], and can be purchased in bulk for roughly $400 - $500/KWh [2,10]. In contrast, smaller storage units have life cycles in the hundreds and cost tens of dollars per kWh [4]. Specific details of our storage cost model is given in Section 4.4 because it depends on other details of our model.

### 3.4 Goals

Our goal is twofold. First, given that everyone benefits if the load factor is reduced, we wish to *select a pricing scheme for Ontario such that when agents (homeowners with storage) greedily use their storage to benefit themselves given this pricing scheme, they also minimize the load factor*. Second, we find whether storage is profitable for homeowners under the various pricing schemes.

We investigate this mechanism design problem in a game-theoretic fashion as follows. We first detail a set of possible pricing schemes we investigate this mechanism design problem in a game-theoretic fashion as follows. We first detail a set of possible pricing schemes we investigate this mechanism design problem in a game-theoretic fashion as follows. We first detail a set of possible pricing schemes we investigate this mechanism design problem in a game-theoretic fashion as follows. We first detail a set of possible pricing schemes.

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<tr>
<th>Name</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A = {1, \ldots, n}$</td>
<td>Set of agents</td>
</tr>
<tr>
<td>$H_O$</td>
<td>Number of homeowners in Ontario</td>
</tr>
<tr>
<td>$\mathcal{H} = {1, \ldots, H_O}$</td>
<td>Set of homeowners</td>
</tr>
<tr>
<td>$I$</td>
<td>Window size</td>
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<tr>
<td>$w_i = {j \mid j = i - 1}$</td>
<td>Optimization window at time $i$ (set of $I$ intervals starting at $i$)</td>
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<tr>
<td>$e_a$</td>
<td>Capacity of agent $a$’s storage</td>
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<tr>
<td>$\alpha^a$</td>
<td>Agent $a$’s storage efficiency</td>
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<tr>
<td>$l^a_i$</td>
<td>Agent $a$’s demand during $i$</td>
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<td>$b^a_i$</td>
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<td>$b^a_i^+$</td>
<td>Agent $a$’s charging profile during $i$</td>
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<td>$b^a_i^-$</td>
<td>Agent $a$’s discharging profile during $i$</td>
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<tr>
<td>$q^a_i$</td>
<td>Agent $a$’s total load during $i$</td>
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<td>$H$</td>
<td>Aggregate homeowners’ load during $i$</td>
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<td>$o_i$</td>
<td>Aggregate Ontario load at time $i$</td>
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<td>$p_i$</td>
<td>Electricity price during $i$ under RTAP (Section 6.3)</td>
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<tr>
<td>$p^i_t$</td>
<td>Agent $a$’s electricity price during $i$ under PRTAP (Section 6.4)</td>
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<tr>
<td>$s$</td>
<td>Distribution of $q_i$ vs. $p_i$ described in Sections 6.3 and 6.4</td>
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### 4. GAME THEORETIC MODEL

We now define our game theoretic formulation of the problems defined in the last section. In Section 4.1, we define our notation. In Section 4.2, we state our assumptions. We define the optimization algorithm agents use to compute their best response to various pricing strategies in Section 4.3. In Section 4.4, we describe how we evaluate the profitability of storage. We describe our dataset in Section 4.5. We describe how we determine the effect of storage adoption on the Ontario load factor in Section 4.6. Finally, we describe how we estimate the size of Ontario’s residential sector in Section 4.7.

#### 4.1 Notation

All notation we use is summarized in Table 1: it is adapted from Vytelingum et al. [31]. We let $A$ represent the set of $n$ agents, which represent homeowners with storage. We use homeowner when referring to Ontario residents in general, and agent when we refer to those residents with storage. Each agent $a \in A$ has a storage capacity $e^a$, a storage efficiency $\alpha^a$, a maximum storage charging rate $b^a_i^+$, and a maximum storage discharging rate $b^a_i^-$. We divide time into hourly intervals. During each hourly interval $i$, each agent has an energy requirement for the hour known as their load profile, $l^a_i$, a storage discharging profile stating the amount of energy the agent discharges from their storage during $i$, $b^a_i^-$, and a storage charging profile stating the amount of energy the agent charges their storage during $i$, $b^a_i^+$. We use $b^a_i = b^a_i^+ + b^a_i^-$ to represent agent $a$’s storage profile during $i$. Each agent $a$’s strategy is to pick values for $b^a_i$ over $i$ as to reduce their total costs.

Throughout this paper, we use the notation $\hat{x}$ to refer to a prediction of a variable $x$.

#### 4.2 Assumptions

In this work we make the following assumptions:

1. Agents cannot sell electricity back to the grid.
2. Agents only alter their storage profiles; they do not change their loads.
3. All agents have identical storage units.
4. At any given interval $i$, agents know the Ontario aggregate load prior to $i$ (this information is published hourly in Ontario), but must predict it for $i$ and future intervals (Section 5.1).
5. At any given interval $i$, agents know their load prior to and including $i$, but must predict it for future intervals (Section 5.2).
6. Agents can estimate their own load for any future interval $i$ on a coarse grained “low/medium/high” scale, where low is less than 300Wh and high is greater than 1.5kWh.
7. Under nondeterministic pricing schemes, at any given interval $i$, agents know the price of electricity prior to $i$ but must predict it for $i$ and future intervals (Section 5.3).
8. Agents are greedy; they do not care about the load factor—they only care about saving money under the pricing scheme.
imposed on them.

9. Given that smart meters are required for some pricing schemes, we assume the grid can separately measure the aggregate load of industry and homeowners.

### 4.3 Optimization Algorithm

Each agent can compute their best response storage profile to all pricing schemes we study by solving the following optimization problem, derived from Vytelingum et al. [31]. We make three changes to their optimization program. First, we use a sliding optimization approach as discussed in this section. Second, we remove the constraint that agents must have a net storage profile of zero during any window because it is suboptimal—see Corollary 1 in the Appendix. Finally, we remove the cost to use storage from the objective function, explained in detail in Section 4.4.

**Best Response Agent Program** [31]:

Objective:

\[
\min \left( \sum_{i \in w_i} p_i \left( b_i^{w_i} - b_i^{w_i} + l_i^w \right) \right)
\]

Subject To:

\[
b_i^{w_i} - b_i^{w_i} \leq b_i^{w_i} + b_i^{w_i} \quad \forall i \in w_i
\]

\[
b_i^{w_i} - \alpha \sum_{j=1}^{i-1} (b_j^{w_i} - b_j^{w_i}) \quad \forall i \in w_i
\]

\[
b_i^{w_i} - \epsilon \sum_{j=1}^{i-1} (b_j^{w_i} - b_j^{w_i}) \quad \forall i \in w_i
\]

\[
b_i^{w_i} - l_i \quad \forall i \in w_i
\]

The agent’s best response is to minimize their total electricity cost over each window, objective (1)\(^1\). Constraint (2) states agents cannot charge or discharge more than the maximal rate for their storage unit. Constraints (3) and (4) ensure that agents cannot discharge more energy than is in their storage, and cannot charge their storage past the storage capacity, respectively. Constraint (3) also enforces storage efficiency: agents can only discharge \(\alpha X\) after charging their storage with \(X\)kWh. Finally, constraint (5) states that an agent’s aggregate electricity usage during any interval cannot be negative\(^2\).

In our model, each agent uses a *sliding optimization window* to estimate their optimal storage profile. We denote the optimization window at time \(i\) as \(w_i = \{i, \ldots, i + I - 1\}\). This differs from the approach of Vytelingum et al. [31], who compute storage profiles using disjoint optimization windows. The difference in the two approaches is depicted in Figure 3. In their model, each agent performs the optimization only at the start of each day (i.e., at the start time intervals of the form 24\(k\), with \(k = 0, 1, \ldots\) and a window of size 24 hours, because they assume agents daily load profiles do not change (they assume only prices vary). At the start of each day, each agent computes strategies for that day, \((\tilde{b}^{24k_i}_i, \tilde{b}^{24k_i+1}_i, \ldots, \tilde{b}^{24k_i+23}_i)\).

\(^1\)We note \(p_i\) in this objective is a parameter set based on the pricing scheme, not an optimization variable. This is important for our simulations because \(p_i\) can be determined prior to running the optimization.

\(^2\)We can remove this constraint in future work to allow for reselling of energy back to the grid. We do not consider this further in this paper due to the complex issues raised by bidirectional electrical flows in the distribution grid.

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### 4.4 Storage Cost

Vytelingum et al. treat the cost to use storage as constant and do not include this cost in their storage profitability analysis. In this section we describe how we include the cost of storage in our storage profitability analysis.

In modern day Li-ion (Lithium-Ion) batteries, depth of discharge does not have a significant impact on the life of the battery, but the battery needs to be replaced after a number of cycles; for Li-ion this is between 400-500 cycles [4]. A cycle is defined to be completed when the battery changes from a period of discharging/idle to a period of charging. Interestingly, the cost to discharge a 4kWh unit of storage by \(x\)kWh \((x \leq 4)\) then recharge it is the same for all \(x\).

We study two classes of optimization problems: those that do and do not include storage depreciation costs in the optimization objective function. When cycle costs are included, objective function (1) needs to be modified. In the objective, a cost should be incurred whenever the agents charges profile changes from nonpositive to positive. Let \(p_{c}\) represent the degradation cost of one cycle. Then the objective is

\[
\min \left( \sum_{i \in w_i} p_i \left( b_i^{w_i} - b_i^{w_i} + l_i^w \right) + C_{p_{c}} \right)
\]

\[
I_i = \begin{cases} 
1 & b_i^{w_i} > 0 \land b_i^{w_i} \leq 0 \\
0 & \text{otherwise} 
\end{cases}
\]

This modified objective is an integer program (IP), which are generally much harder to solve than linear programs (LPs). Therefore, we use the cycle-cost oblivious LP. If the average number of cycles per day was large, this would be a poor solution to the problem. However, we find agents do not cycle storage more than a few times a day under the LP, corresponding to daily demand peaks. Therefore, after the simulations we add back a cost to the agents of \$20/kWh every 500 cycles [4]. This allows us to approximate if...
storage was profitable for the agents.

4.5 Measured Home Data
For our simulations we use real data from Ontario homes [1]. We have measured the power consumption of 16 homes every six seconds for five to nine months. We aggregate this data into hourly intervals for this research.

4.6 Evaluating the Change In Load Factor
To determine how the introduction of home storage affects the Ontario load factor, we estimate the load factor assuming a percentage of homeowners have storage, and compare it to the real load factor. Let \( \omega \) be the penetration rate, the proportion of Ontario homes with storage, and \( \beta \) represent the fraction of the Ontario aggregate contributed by the residential sector. We estimate the new aggregate load with respect to \( \omega \) as

\[
\hat{o}_i(\omega) \approx (1 - \beta) o_i + (1 - \omega) \beta o_i + \beta \omega o_i \frac{q_i}{l_i}
\]

The first term in the estimation is the contribution to the Ontario aggregate load from industry. The second term is the contribution from homes without storage. The last term approximates the contribution from homes with storage. We suppose that as \( n \) approaches \( \omega H_0, l_i \) approaches \( \beta \omega o_i \) for all \( i \). We thus use a factor of \( \frac{1}{n} \) to scale this proportion of the aggregate load to account for the introduction of storage. In Ontario, \( \beta \approx 0.3 \) [26].

4.7 Estimating the Size of the Residential Sector
Until now we have abstracted the complexity of the residential sector, stating homes contribute to 30% of the Ontario aggregate load. More accurately, homes and small businesses not classified as commercial or industrial represent 30% of the aggregate. To estimate \( H_0 \), we need the number of units in this sector, but since we do not have recent data for this number, we estimate it as the average of two different approximations as follows:

1. The last census for which data is available at the time of this writing in Ontario was in 2006. In 2006, there were \( \approx 4.6 M \) households in Ontario [23]. This was an 8% increase from 2001. Assuming the same rate of growth, we estimate the number of households in 2011 as \((1 + 0.08) \cdot 4.6 \approx 5 M \). In 2008, there were \( \approx 900,000 \) million small businesses in Ontario [15]. This gives a rough approximation to the size of the residential sector in 2011 as \( 5.9 M \).

2. The average load of our measured homes without storage at each hour \( i \) for our dataset is \( l_i/n \). Let \( D \) represent the length of our dataset in hours. At any time \( i \), \( \frac{\sum o_i}{l_i/n} \) approximates the number of homes assuming the average home load of our agents is similar to the Ontario average. We approximate the number of homes averaging this quantity as follows:

\[
\frac{1}{D} \sum_{i=0}^{D-1} \beta o_i \left( \frac{l_i}{n} \right)
\]

This estimates \( H_0 \) as \( 6.3 M \) units.

We assume \( H_0 \) is the average of these two estimates, \( 6.1 M \).

Intuitively, \( \beta \omega o_i \) is the proportion of the aggregate load contributed by agents in \( A \), prior to the introduction of storage.

5. PREDICTION
In this section we describe our prediction algorithms.

5.1 Aggregate Load Prediction
We use a MATLAB feed forward neural network fitting model for Ontario aggregate load prediction. We use a decade’s worth of hourly aggregate load data for training [13]. To predict the load for a given hour \( i \), the predictor uses the date and time of \( i \), the outside temperature at \( i \), and whether \( i \) falls on a weekend or holiday. We denote agents’ estimates of the Ontario aggregate load for \( w_i \) as \( (\hat{o}_i, ..., \hat{o}_{i+f-1}) \). Predicting aggregate load is easier than predicting personal loads because variations are smoothed when aggregated; our predictor for the Ontario aggregate has a mean relative error of about 4%. Predictions for an arbitrary time period are shown in Figure 4.

Our current prediction models do not take into account change in aggregate load as a result of storage. For small penetration rates, the aggregate load with storage will be similar to the aggregate load without storage. However, as the penetration rate increases, our predictor will suffer from a larger error.

5.2 Individual Home Load Prediction
We use the same type of MATLAB predictor for individual home load prediction. In addition to the time, temperature and whether or not it is a weekend or holiday, the predictor uses a “high/medium/low” indicator stating whether the agent’s load for the hour to be predicted is less than 300Wh, greater than 300Wh but less than 1.5kWh, or greater than 1.5kWh. We denote agent \( a \)’s estimates for window \( w_i \) as \( (\hat{l}_{i}, ..., \hat{l}_{i+f-1}) \). The mean relative errors of our individual home load predictors range from 10% to 30%.

5.3 Price Prediction
For two of the pricing schemes we evaluate, agents need to predict electricity prices because they are nondeterministic. Prices for both of these strategies depend on the relationship between aggregate load and the HOEP. We use a best-fit linear regression model to find this relationship, trained once more by a decade’s worth of hourly data [13]. We found a linear model was sufficient; our mean relative error in predicting the HOEP is about 5%.
6. PRICING STRATEGIES

In this section, we detail some possible pricing schemes and describe the agents’ best responses to the proposed strategy. We further explain how we simulate the pricing strategy to determine its effect on the load factor.

6.1 Tiered Usage-Based Pricing (TUBP)

In Ontario, there are currently two electricity pricing schemes deployed. Customers that are not yet monitored by smart meters are charged under Tiered Usage-Based Pricing (TUBP) [20]. Under TUBP, homeowners pay a price for each kWh of energy used during each month, \( p_i > p_0 \) for the first \( x_0 > 0 \) kWh of energy used during each month, \( p_i > p_0 \) for the next \( x_1 > 0 \) kWh, and so forth. In other words, energy becomes more expensive as homeowners consume more.

Agents compute their best response to TUBP using the algorithm defined in 4.3 and setting \( p_i \) in objective (1) deterministically as follows. Let \( m_a^i \) represent the amount of energy \( a \) has received since the beginning of the month \( i \) is in. Then

\[
p_i = \begin{cases} 
0.0071/kWh & \text{if } m_a^i < 1000 \\
0.0083/kWh & \text{otherwise}
\end{cases}
\]  

rather than as a function of aggregate load [20].

Starting with some predefined interval, we simulate TUBP as follows. For each interval \( i \),

1. Each agent predicts their own load for \( w_i \) {\( a \)} = \{\( \tilde{t}_{i+1}, \ldots, \tilde{t}_{i+I-1} \) \).
2. Each agent uses these price predictions to solve their best response to compute their charging profile \( \{\tilde{b}_a^i, b_a^i, \ldots, b_a^{i+I-1} \} \), using the algorithm given in Section 4.3.
3. The agents’ aggregate load \( q_i \) for time \( i \) is calculated as \( \sum_a \tilde{t}_a^i + \sum_a b_a^i \). This is needed to determine the effect of storage on the load factor.

6.2 Time of Day Pricing (TODP)

As smart meters are deployed in Ontario, homeowners are starting to pay for electricity under time of day pricing (ToDP) [20]. In this pricing scheme there are four daily time slices. In the summer (May-October inclusive), there is one off-peak period where electricity is cheapest, two mid-peak periods, and one on-peak period during which electricity is most expensive. The winter scheme is similar, but there are two peak periods and only one mid-peak period. Both schemes are depicted in Figure 5. We denote \( p_{\text{min}} < p_{\text{mid}} < p_{\text{max}} \) as the prices of electricity during these periods.

Agents compute their best response to TUBP using the algorithm defined in 4.3 and setting \( p_i \) in objective (1) deterministically based on Figure 5 rather than as a function of aggregate load [20].

We simulate this pricing strategy as in TUBP.

6.3 Real Time Aggregate Pricing (RTAP)

Vyteltingum et al. [31] examined the effect of real time aggregate pricing (RTAP) on various social welfare factors based on data from the UK. Under this strategy, the price per unit of electricity all homeowners pay is \( p_i = s(q_i) \), where \( q_i \) is the Ontario aggregate load, and \( s \) is the distribution of aggregate load vs. market price of electricity. We compute \( s \) as discussed in Section 5.3. We note that in the UK, as in Ontario, this pricing method is not currently used. This pricing method requires fine-grained measurement of homes’ electricity consumption; measurements that will not be available until the mass deployment of smart meters.

Agents compute their best response to TUBP using the algorithm defined in 4.3 and setting \( p_i \) in objective (1) as just discussed.

Starting with some predefined interval, we simulate RTAP as follows. For each interval \( i \),

1. Each agent predicts their own load for \( w_i \) {\( a \)} = \{\( \tilde{t}_{i+1}, \ldots, \tilde{t}_{i+I-1} \) \).
2. We predict the Ontario aggregate load for \( w_i \) {\( \tilde{o}_i \) \}(\( \tilde{o}_i, ... , \tilde{o}_{i+I-1} \)). This prediction is common to all agents.
3. Using the aggregate load predictions and \( s \), we predict the price of electricity for \( w_i \) {\( \tilde{p}_i \) \}(\( \tilde{p}_i, ... , \tilde{p}_{i+I-1} \)). This prediction is common to all agents.
4. Each agent uses these price predictions to solve their best response to compute their charging profile \( \{\tilde{b}_a^i, b_a^i, \ldots, b_a^{i+I-1} \} \), using the algorithm given in Section 4.3.
5. The agents’ aggregate load \( q_i \) for time \( i \) is calculated as \( \sum_a \tilde{t}_a^i + \sum_a b_a^i \). This is needed to determine the effect of storage on the load factor.

6.4 Proportional Real Time Aggregate Pricing (PRTAP)

RTAP is unfair in the following sense. Let \( H_{\text{RTA}} \) denote the total amount of money the distributors receive from homeowners under RTAP during interval \( i \). We can see \( H_{\text{RTA}} \) is proportional to each homeowners’ load, but the price per unit each homeowner pays is not proportional to their load; all homeowners pay the same price per unit—\( s(q_i) \). Consider two homeowners, \( A \) who suffers from a large demand during interval \( i \), and \( B \) who has a low demand during time \( i \). Both homeowners pay \( s(q_i) \) per unit, but this is unfair to \( B \); the cost of fulfilling \( B \)’s load is high due to no fault of its own!

To solve this problem, we propose another strategy we call proportional real time aggregate pricing (PRTAP). We note that deriving this new pricing function \( s'(q_a, q_i) \) is a design challenge in itself, and in future work we will investigate possibly better functions than the one we choose here.

![Figure 5: Current Ontario TOD prices [20]. At the time of writing, peak prices are 10.8cents/kWh, mid-peak is 9.2cents/kWh, and off-peak is 6.2cents/kWh. All weekends are off-peak.](image-url)
For now, we assume the total the distributors receive, \( H_{\text{RTA}} \), cannot change, yet we want the price each homeowner pays per unit to be proportional to their own load; the higher their contribution to the aggregate load, the higher their price. We derive the price each homeowner \( h \in \mathcal{H} \) pays per unit \( p_i^h \) under PRTAP as:

\[
p_i^h = \left( \frac{t_i^h}{\sum_{k \in \mathcal{H}} (t_k^h)^2} \right) H_{\text{RTA}} \tag{10}
\]

Then,

\[
\sum_{k \in \mathcal{H}} t_i^h p_i^h = \sum_{k \in \mathcal{H}} t_i^h \left( \frac{t_i^h}{\sum_{k \in \mathcal{H}} (t_k^h)^2} \right) H_{\text{RTA}} \tag{11}
\]

\[
= \frac{1}{\sum_{k \in \mathcal{H}} (t_k^h)^2} H_{\text{RTA}} \sum_{k \in \mathcal{H}} (t_i^h)^2 = \sum_{k \in \mathcal{H}} (t_k^h)^2 H_{\text{RTA}} \tag{12}
\]

\[
= H_{\text{RTA}} \tag{13}
\]

Therefore, each homeowner now pays a price per unit that is proportional to their contribution to the aggregate load, and the distributors still receive the same amount of money. Intuitively this seems to be a fairer pricing scheme than RTAP.

Agents compute their best response to PRTAP using the algorithm defined in 4.3 and setting \( p_i \) in objective (1) with \( p_i \) from Equation (10).

We need to make an additional assumption to simulate PRTAP. Agents do not know \( \sum_{k \in \mathcal{H}} (t_k^h)^2 \) in Equation (10). Let \( H_O \) be the number of homes in Ontario, and \( \beta \) represent Ontario homes’ contribution to the Ontario aggregate load. We approximate \( \sum_{k \in \mathcal{H}} (t_k^h)^2 \) using the equation:

\[
H_O \left( \frac{\beta a_i}{H_O} \right)^2 \tag{16}
\]

This prediction also suffers from a problem similar to the one discussed at the end of Section 5.1; as storage penetration is increased, agents’ predictions of the aggregate load will be more erroneous.

We approximate \( H_{\text{RTA}} \) (the amount of money received under RTAP) similarly; agents do not know this value because it depends on other agents’ loads. Let \( P_{\text{RTAP}} \) represent the price during \( i \) assuming RTAP. Agents approximate \( H_{\text{RTA}} \) as \( P_{\text{RTAP}} \cdot \beta a_i \), which approximates the amount of money paid by from homeowners under RTAP.

Though computing the best response for PRTAP is similar to RTAP, the simulation strategy is modified as follows. Starting with some predefined interval, for each interval \( i \),

1. Perform steps 1 and 2 of the RTAP simulation.
2. Given the Ontario aggregate predictions, we predict the aggregate load of homeowners as \( \beta a_1, ..., \beta a_{i-1} \). This prediction is common to all agents.
3. Using their personal prediction, \( a_i \), and Equation (10), each agent predicts the price they pay for \( w_i, (p_i^1, ..., p_i^{i-1}) \).
4. Perform steps 4 and 5 of the RTAP simulation.

### 6.5 Extreme Pricing (EXTREME)

![Figure 6: The percent change in average cost between storage and storageless agents over a period of approximately 6 months when each agent with storage has a capacity of 4 kWh. Concretely, for each pricing scheme \( p \), this is calculated as \( 100 \cdot \sum_{a \in \mathcal{A}} (C^s_a - C_A) / \sum_{a \in \mathcal{A}} C_{A_a} \), where \( C^s_a \) is agent \( a \)'s total cost with storage under \( p \) and \( C_A \) is agent \( a \)'s total cost without storage under \( p \). A positive value can be interpreted to mean that agents incur a greater cost (on average) when storage is adopted. Note that storage is only profitable in the EXTREME pricing scheme.

We evaluate an additional scheme, EXTREME, where electricity is \$0.02 per kWh from 7pm to 7am and ten times as expensive from 7am to 7pm. We compute the agents best responses to this pricing scheme and simulate this pricing scheme as done for TODP.

### 7 EVALUATION

We now describe the results of our simulations.

#### 7.1 Profitability of Storage

We measure the total revenue paid by each agent to the grid with and without storage. The results (assuming a storage size \( e_a = 4 \text{kWh} \) and including the cost incurred by storage as discussed in Section 4.4) are shown in Figure 6. We can see that when we include the cost of storage, homeowners pay more with than without storage for all pricing schemes excluding EXTREME. We note that the EXTREME scheme was designed specifically to demonstrate storage can be profitable under sufficient conditions. We conclude that under schemes in use, such as TOD and TUBP, agents will not adopt storage for this purpose unless the cost of storage is subsidized—homeowners will not adopt storage if it is not profitable.

#### 7.2 Load Factor

We find that the introduction of storage can, in fact, increase the load factor. Under pricing schemes that incentivize charging at times known to all agents, the load factor will increase rather than decrease at high storage penetration levels. As a thought experiment, suppose the penetration rate is 1 and that electricity is cheapest at hour \( h \) each day. If agents’ storage capacities are sufficiently large, a new peak arises at \( h \)—all agents observe the cheapest price at \( h \), and because they use the same optimization model, all agents’ best responses will be to charge at \( h \). This new peak at \( h \) resulting from
correlated charging increases the load factor. A similar problem has been observed in networking literature [9].

Figures 9, 10 and 11 show the load factor for various penetration rates. We find that for a given capacity, the load factor decreases to a minimum, which occurs at penetration rate $\omega^*$, and then begins increasing monotonically. Furthermore, there exists a penetration level $\omega^H \geq \omega^*$ for each pricing scheme where the load factor at penetration levels greater than $\omega^H$ is worse than the load factor at $\omega = 0$; we recognize this as the point at which the grid is negatively affected by higher levels of storage adoption. Table 2 lists values for $\omega^*$ and $\omega^H$ under the different pricing schemes when $e = 4\text{kWh}$.

Vytilingum et al. [31] claim that the load factor under RTAP converges to approximately 1.064 as homeowners adopt storage, but our results differ. Figures 9 and 10 indicate a significant change in the behaviour of the load factor under RTAP as the capacity increases from 4kWh to 8kWh. Examining the surface in Figure 8, we note a pronounced increase in the RTAP load factor for storage capacities less than $\approx 5\text{kWh}$. We find that with capacities greater than or equal to 5kWh, agents have enough storage capacity to satisfy their loads throughout most peak periods. However, with less storage, agents may need to charge at a local minimum during the peak period. At high penetration levels, many agents may be charging during this local minimum, which induces a peak that may increase the load factor. This is depicted in Figure 7.

We note that our results will become more accurate as the size of our dataset increases, as discussed in Section 8. Regardless of the small size of our dataset, we believe our methodology and the trends highlighted by our data are sound. The reader should note that as highlighted in Section 5, higher penetration rates will yield less accurate results. We are thus more interested in trends visible at low penetration rates.

Interestingly, we find that PRTAP, although designed to be a more fair pricing scheme, has a negative effect on the load factor. We discuss this further in Section 8.

7.3 Metagame

\footnotetext{A peak in the homeowners’ aggregate load at time $h$ does not necessarily create a new aggregate peak at $h$ if the $e^a$ are small, because, without storage, industry still represents 70% of the aggregate. However, as $e^a$ increaseS, if all agents charge at $h$ then the probability that the peak load is shifted to $h$ goes to 1. Intuitively, as $e^a$ increases, homes represent a larger percentage of the aggregate load at time $h$.}

Figure 7: High levels of storage penetration may correlate otherwise uncorrelated (or weakly correlated) loads and thus create a larger peak.

Figure 8: The load factor for varying levels of penetration $\omega$ and varying amounts of storage capacity $e^a$ under RTAP. Given a constant storage capacity and small enough levels of penetration, the load factor decreases.

<table>
<thead>
<tr>
<th>$\omega^*$</th>
<th>PRTAP</th>
<th>RTAP</th>
<th>TOD</th>
<th>TUBP</th>
<th>EXTREME</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega^H$</td>
<td>0.00875</td>
<td>0.05125</td>
<td>0.08375</td>
<td>0</td>
<td>0.25875</td>
</tr>
<tr>
<td></td>
<td>0.01125</td>
<td>0.08625</td>
<td>0.09875</td>
<td>0</td>
<td>0.34875</td>
</tr>
</tbody>
</table>

Table 2: Approximations to the penetration level threshold values for $e = 4\text{kWh}$ with a margin of error of $\pm 0.00125$.

Our evaluation demonstrates that pricing schemes may merely shift peaks rather than reduce them. After the peak has shifted, the pricing scheme will need to be modified so that the cheapest electricity price is not charged when the load is highest. In the Ontario energy market, this corresponds to the reevaluation of the RPP as outlined in Section 3.1. However, the introduction of a new pricing scheme may suffer from the same shortcomings—agents will again adapt their storage profiles according to the new price and we are faced with the same problem of having to reevaluate the pricing scheme.

We identify this as a metagame being played between homeowners and regulators. The regulators are tasked with finding a pricing scheme that minimizes the load factor. Because their strategy is a best-response to the behaviour exhibited by homeowners, and each homeowner’s best-response is also a function of the pricing scheme implemented by regulators, we encounter the cyclic phenomenon described above when the penetration becomes sufficiently large. One naive solution to this problem is the employment of a flat pricing scheme that does not incentivize charging storage at any time. However, under a flat pricing scheme, storage has no benefit for the grid and negatively affects all homeowners because they can no longer recoup their investment costs.

8. FUTURE WORK

Here we detail extensions to our work that can improve the accuracy of our results. We are working to measure more homes and hope to revisit the problem with a larger dataset.

1. Currently, the predictors employed by agents decrease significantly in accuracy as $\omega$ grows because they are not observing the change in aggregate load as feedback. As discussed in Section 5.1, agents in our simulation predict the aggregate load using past Ontario aggregate load (where homeowners do not have storage). As the duration of our dataset grows,
penetration, $\omega$, with constant capacity $e = 4kWh$.

we can slowly introduce storage to the agents and retrain the aggregate load predictor over time. This would allow us to simulate higher values of $\omega$ with higher confidence.

2. We would have a more realistic simulation in terms of storage maintenance costs by using objective (6), but performing our simulations using this integer program may be computationally intractable.

3. We have identified the existence of a metagame between the agents and the entities responsible for modifying the pricing scheme in Section 7.3. We would like to identify values of $\omega$ at which this game becomes chaotic under the pricing schemes discussed in this work. We would also like to investigate possible pricing schemes that do not suffer from this cyclic property (other than the trivial flat pricing scheme).

4. We identified that RTAP is not a fair pricing strategy, and proposed a fairer method, PRTAP, that generates the same amount of money. However, our proposed strategy degrades the load factor significantly. The objective of discovering a pricing scheme that is both fair (i.e. the price per unit paid is proportional to the energy drawn) and does not degrade the load factor is a point we wish to investigate.

Figure 10: The load factor as a function of the level of storage penetration, $\omega$, with constant capacity $e = 8kWh$.

Figure 11: The load factor as a function of the level of storage penetration, $\omega$, with constant capacity $e = 16kWh$.

9. CONCLUSIONS

Our results come as a surprise and are contradictory to the closest related work. We conclude that while we found storage to be profitable for the consumer under some pricing schemes, we found storage may increase peak load rather than decrease peak load. Perhaps most interestingly, we found that for sufficiently high penetration rates, pricing schemes that incentivize charging at times known to all agents will correlate otherwise uncorrelated (or weakly correlated) loads and thus increase the peak load.

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Appendix

Theorem 1: RTAP/PRTAP Infinite Optimization Window

The optimal optimization RTAP/PRTAP window is infinite. That is, cases exist where computing a charging strategy $(b_i^w)_{i=1}^{w=\infty}$ using a window with finite length $w > 0$ is suboptimal.

PROOF. The time to fully charge $a$’s storage from empty is $h^+ = \frac{e^a}{P_i}$. The time to fully discharge $a$’s storage from full is $h^- = \frac{e^a}{P_i}$. To simplify the proof, we assume that $h^+, h^- \in \mathbb{Z}$. The extension to $\mathbb{R}$ complicates notation but can be shown similarly.

Suppose, without loss of generality that there exists a window of finite length $w \in \mathbb{N}$ with $w > h^+ + h^- \footnote{If a window $w$ yields suboptimal results, so will any smaller window. Hence, proving the case for an arbitrarily large window also proves the theorem for any smaller window.}$ and that it yields an optimal charging strategy.

For convenience we denote $(a, b) \cap \mathbb{Z}$ (the integers lying between $a$ inclusive and $b$ exclusive) as $[a, b)$. Suppose that the price of electricity is

$$p_i = \begin{cases} P_0 & i \in [0, h^+) \\ P_1 & i \in [h^+, h^+ + h^-) \\ P_2 & i \in [h^+ + h^-, \infty) \end{cases}$$
where $0 < P_0 < P_1 < P_2$, and that agent $a$’s load factor is

$$l_i^a = \begin{cases} 0 & i \in [0, h^+) \\ b_i^a & i \in [h^+, h^+ + h^-) \\ 0 & i \in [h^+ + h^-, h^+ + h^- + w - 1) \\ b_i^a & i \in [h^+ + h^-, w - 1, \infty) \end{cases}$$

This is depicted in Figure 12.

If the agent computes their optimal strategy at any time in the interval $[0, h^+ + h^-)$, they will not observe the demand at time $h^+ + h^- + w - 1$. Their optimal strategy computed at any point in this interval is to fully charge their storage while the price is $P_0$, fully discharge it while the price is $P_1$, and neither charge nor discharge while the price is $P_2$. In other words, if $t \in [0, h^+)$, $b_t^a = b_t^a$, if $t \in [h^+, h^+ + h^-)$ $b_t^a = b_t^a$, and if $t \in [h^+ + h^-, t + w)$, $b_t^a = 0$.

At time $h^+ + h^- - w$, the agent is now in the interval where the price is $P_2$ and they can observe the demand at time $h^+ + h^- + w - 1$. Since the agent discharged the last of their storage at time $h^+ + h^- - 1$, the agent has no storage left. Hence, they must pay for the demand at time $h^+ + h^- + w - 1$.

In this case, the total price the agent pays is $P_0 e^{a} + P_2 b_t^a$. However, if at time $h^+ + h^- - w$ the window size was $w + 1$, they would have deferred discharging their remaining storage until time $h^+ + h^- + w$. In this case, the agent would have paid $P_0 e^{a} + P_2 b_t^a < P_0 e^{a} + P_2 b_t^a$. Hence if the window size was $w + 1$, the agent could have computed a better strategy; which contradicts that the agent can always compute an optimal strategy using a window size of $w$.

\[ \square \]

**Theorem 2: Restricted RTAP/PRTAP Infinite Optimization Window**

The optimal optimization RTAP/PRTAP window is infinite even with an upper bound on the price of electricity, $P_{\max} > 0$ and a lower bound on agent $a$’s load, $L_{\min} > 0$. That is, cases exist where computing a charging strategy $(b_t)_{t=m,w}$ using a window with finite length $w > 0$ is suboptimal.

**Proof.** $h^+, h^-$ and $w$ are defined as in Theorem 9.

Suppose that the price of electricity is

$$p_t = \begin{cases} P_{\max} - 3\epsilon & i \in [0, h^+) \\ P_{\max} - \epsilon & i \in [h^+, h^+ + h^-) \\ P_{\max} - 2\epsilon & i \in [h^+ + h^-, h^+ + h^- + w - 1) \\ P_{\max} & i \in [h^+ + h^-, h^+ - w - 1, \infty) \end{cases}$$

where $\epsilon > 0$, and that agent $a$’s load factor is

$$L_{t}^a = \begin{cases} L_{\min} & i \in [0, h^+) \\ L_{\min} + b_t^a & i \in [h^+, h^+ + h^-) \\ L_{\min} & i \in [h^+ + h^-, h^+ + h^- + w - 1) \\ L_{\min} + b_t^a & i \in [h^+ + h^-, h^+ - w - 1, \infty) \end{cases}$$

If the agent computes their optimal strategy at any time in the interval $[0, h^+ + h^-)$, they will not observe the demand at time $h^+ + h^- + w - 1$. Their optimal strategy computed at any point in this interval is to fully charge their storage while the price is $P_{\max} - 3\epsilon$, fully discharge it while the price is $P_{\max} - \epsilon$, and neither charge nor discharge while the price is $P_{\max} - 2\epsilon$. In other words, if $t \in [0, h^+)$, $b_t^a = b_t^a$, if $t \in [h^+, h^+ + h^-)$ $b_t^a = b_t^a$, and if $t \in [h^+ + h^-, t + w)$, $b_t^a = 0$.

At time $h^+ + h^-$, the agent is now in the interval where the price is $P_{\max} - 2\epsilon$ and they can observe the demand at time $h^+ + h^- + w - 1$. Since the agent discharged the last of their storage at time $h^+ + h^- - 1$, the agent has no storage left. Hence, they must pay for the demand at time $h^+ + h^- + w - 1$.

In this case, the total price the agent pays is $(P_{\max} - 3\epsilon + L_{\min}) + (P_{\max} - \epsilon + L_{\min}) + (P_{\max} - 2\epsilon + L_{\min}) + (P_{\max} - \epsilon + L_{\min})$. However, if at time $h^+ + h^- - 1$ the window size was $w + 1$, they would have deferred discharging their remaining storage until time $h^+ + h^- + w$. In this case, the agent would have paid $(P_{\max} - 3\epsilon + L_{\min}) + (P_{\max} - \epsilon + L_{\min}) + (P_{\max} - \epsilon + L_{\min}) + (P_{\max} - \epsilon + L_{\min})$. Hence if the window size was $w + 1$, the agent could have computed a better strategy; which contradicts that the agent can always compute an optimal strategy using a window size of $w$.

\[ \square \]

**Corollary 1: Net Storage of Zero In One Window**

Requiring agents to have a net storage profile of zero in each interval is suboptimal.

We showed in the previous two theorems there are cases where...
agents are best off charging their storage during one window \(w_1\), saving it, then discharging it in a later window \(w_2\). In this case, the net storage profile for \(w_1\) would not be zero.

**Theorem 3: ToDP Optimization Window**

Under ToDP, given \(h_{\text{OP}}\) as the number of off-peak hours per day and \(c\) as the agent’s capacity, if \(c \leq h_{\text{OP}} \cdot b^a_+\), an optimal optimization window spans from the current time to the start of the off-peak in the next day. Otherwise, the optimal optimization window is infinite.

**Proof.** If \(c \leq h_{\text{OP}} \cdot b^a_+\), the agent can fully charge their storage during a single off-peak period. Thus, there is never a need to save energy from a day \(d\) for use in a future day \(d_F\), since the storage can always be charged fully during the off peak period of \(d_F\).

The proof of the case where \(c > h_{\text{OP}} \cdot b^a_+\) is more involved; we prove it by contradiction. Without loss of generality, we suppose that \(w \in \mathbb{N}\), the length of the optimal optimization window, is measured in days. Further suppose \(w\) is finite. That is, an agent can always compute their best strategy by examining their load under a window of size \(w\). Suppose that the agent has demand

\[
l_i^a = \begin{cases} l_{\text{MIN}} & \text{if } i \in \text{mid peak or off peak} \\ N & \text{if } i \in \text{peak} \end{cases}
\]

for \(w\) days, and

\[
l_i^a = \begin{cases} l_{\text{MIN}} & \text{if } i \in \text{off peak or mid peak 2} \\ l_{\text{MIN}} + R & \text{if } i \in \text{mid peak 1} \\ N & \text{if } i \in \text{peak} \end{cases}
\]

on the \((w + 1)\)th day. We pick \(N\) and arbitrarily small \(R\) satisfying

\[
h_{\text{OP}} \cdot b^a_+ = N + \frac{R}{w + 1} = N + \epsilon \quad (17)
\]

\[
c \geq N + R \geq h_{\text{OP}} \cdot b^a_+ - \frac{R}{w + 1} + R > b^a_+ \cdot h_{\text{OP}} \quad (18)
\]

Figure 14 gives a graphical description. This condition states that the agent can charge a maximum of \(b^a_+ = N + \epsilon\) units of energy during the off-peak on any day. Since they have \(N\) peak demand each day, the agent should store each off peak to fulfill that demand, which means they can charge and carry over \(\epsilon\) energy for at least \(w + 1\) days via Equation (18). However, on day 1, assuming the window size is \(w\), the agent does not see the future demand of \(R\) during the mid peak. Therefore, the agent has no incentive to charge the additional \(\epsilon\) on day 1. However, on day 2, the agent can see the demand of \(R\) on the \((w + 1)\)th day, but at this point the agent can only accumulate \(R - \epsilon\) by that time. Hence, the optimal strategy would have been to charge the additional \(\epsilon\) on day 1, which contradicts the assumption the agent can always compute their optimal strategy if the window size is \(w\).

**Theorem 4: TUBP Infinite Optimization Window**

The optimal optimization TUBP window is infinite. That is, cases exist where computing a charging strategy using a window with finite length \(w > 0\) is suboptimal.

**Proof.** Without loss of generality, we suppose that \(w \in \mathbb{N}\), the length of the optimal optimization window, is measured in months. Further suppose \(w\) is finite.

Suppose that an agent has an aggregate demand of \(x_0 - \epsilon\) for the first \(w\) months. Further suppose that during the \((w + 1)\)-th month, the agent has an aggregate demand of \(x_0 + w\epsilon\). Figure 15 depicts this scenario. During the first hour of the first month, the agent is unable to see the aggregate demand occurring during the \((w + 1)\)-th month and does not employ any storage. Supposing that the "extra" demand \(w\epsilon\) appears only in the last hour of the last month, the agent will pay a total of \(P_0 ([w + 1] x_0 + (w - 1) \epsilon] + P_1 \epsilon\). However, using a window of size \(w + 1\), the total cost becomes \(P_0 ([w + 1] x_0 + w\epsilon]\), which contradicts the assumption that the agent can always compute their optimal strategy if the window size is \(w\).

Note that in the above proof, we do not mention the agent’s max charge rate, \(b^a_+\), its max discharge rate, \(b^a_-\), and its capacity \(c^a\). This is because we can always pick \(\epsilon\) small enough to satisfy the inequalities involving the above constants.

**10. REFERENCES**


